2014년 1학기 공학수학1 중간시험

2014년 4월 21일, 09:45 – 10:45

성명 : **학번 :**

1. (45 pts.) For a 4×8 matrix A, we performed eliminations and got a reduced row echelon form R. For each of the following questions, give and justify your answer.

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 & -1 & 1 & 1 & -2 \\ -3 & -6 & 2 & -7 & 7 & 0 & -6 & 3 \\ 1 & 2 & 2 & 5 & 3 & 3 & -1 & 0 \\ 2 & 4 & 0 & 6 & -2 & 1 & 3 & 0 \end{bmatrix}, R = \begin{bmatrix} 1 & 2 & 0 & 3 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(1) (5 pts.) What is the rank of A ?

(2) (5 pts.) What are the dimensions of $C(A)$, $N(A)$, $C(A^T)$, and $N(A^T)$?

(3) (5 pts.) For a given b, how many solutions does $Ax = b$ have? Does it depend on b ?

(4) (5 pts.) Is the set of rows of A linearly independent ?

(5) (5 pts.) Does the set of columns 4, 5, 6, and 7 of A form a basis of $\mathbf{R}^4$?

(6) (5 pts.) Give a basis of $N(A)$.

(7) (5 pts.) Give a basis of $N(A^T)$.

(8) (5 pts.) Give the reduced row echelon form of A^T.

(9) (5 pts.) Let $R = EA$. Is E invertible ? If so, give E^{-1}.

2. (30 pts.) The following information is known about an $m \times n$ matrix A. For each of the following questions, give and justify your answer.

$$A\begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, A\begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, A\begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}, A\begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

(1) (8 pts.) Prove that the set of four vectors $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ is a basis of $\mathbf{R}^4$.

(2) (8 pts.) Give a matrix C and a nonsingular matrix B such that $A = CB^{-1}$.

(3) (8 pts.) Find a basis for the null space of A^T.

(4) (6 pts.) What are m, n, and the rank of A ?

3. (25 pts.) For each of the following statements, write down **T** if it is true, or **F** otherwise. And explain why by giving a short proof or a counter- example. (No reason, no credit)

(1) (5 pts.) If A is invertible, $C(A^{-1}) = C(A)$.

(2) (5 pts.) Suppose $A \in \mathbf{R}^{m \times n}$, $rank(A) = m < n$. Then there exists a right inverse of A.

(3) (5 pts.) Let $A = uv^T$ where u, v are nonzero vectors in $\mathbf{R}^n$. Then $rank(A) = 1$.

(4) (5 pts.) If $D = AB$ and A is invertible, then $C(D^T) = C(B^T)$.

(5) (5 pts.) For $A \in \mathbf{R}^{m \times n}$, $N(A^T A) = N(A)$.